

| Aufg. | 2003 AI | BE | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|----|------|---|------|---|------|---|---|---|-------------|---|------|---|------|---|------|---|------|---|--|---|--|---|--|---|--|---|--|--|---|
| 1 | $f'(x) = \frac{3}{4}x^2 - \frac{9}{2}x + C_1; f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + C_1x + C_2$ <p>(1) $f(-1) = 0$; (2) $f(0) = \frac{5}{4}$ ergibt</p> $f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 - \frac{5}{4}x + \frac{5}{4}$ | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.1 | $g_a(x) = \frac{1}{4}(x+1)(x^2 - 10x + a) = \frac{1}{4}(x^3 - 9x^2 + ax - 10x + a)$ $g_5(x) = \frac{1}{4}(x^3 - 9x^2 - 5x + 5) = f(x)$ | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.2 | $g_a'(x) = \frac{1}{4}(3x^2 - 18x + a - 10)$. Es gibt keine Extrempunkte, wenn g_a' entweder eine doppelte oder keine Nullstelle besitzt. $D \leq 0 \Leftrightarrow a \geq 37$ | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.3 | $\frac{1}{4}(x+1)(x^2 - 10x + 25) = \frac{1}{4}(x+1)(x-5)^2 = 0 \Leftrightarrow x_1 = -1; x_{2,3} = 5$ | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.4 | $g_{25}'(x) = \frac{1}{4}(3x^2 - 18x + 15); g_{25}''(x) = \frac{1}{4}(6x - 18)$ $g_{25}'(x) = 0 \Rightarrow x_1 = 1; x_2 = 5$ <p>Z.B. mit Hilfe von $g_{25}''(x)$ ergibt sich: H (1 8); T (5 0)</p> | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.5 | $g_{25}''(x) > 0 \Rightarrow x > 3$; $g_{25}''(x) < 0 \Rightarrow x < 3$. Damit ergibt sich: $G_{g_{25}}$ ist linksgekrümmt in $[3; \infty[$ und rechtsgekrümmt in $] -\infty; 3]$. Mit dem Krümmungsverhalten ergibt sich: W (3 4) | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2.6 | (Graph siehe nächste Seite) <table border="1" data-bbox="289 1774 1094 1994" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> </tr> <tr> <td>$g_{25}(x)$</td> <td>0</td> <td>6,25</td> <td>8</td> <td>6,75</td> <td>4</td> <td>1,25</td> <td>0</td> <td>1,75</td> <td>8</td> </tr> <tr> <td></td> <td>N</td> <td></td> <td>H</td> <td></td> <td>W</td> <td></td> <td>T</td> <td></td> <td></td> </tr> </table> | x | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $g_{25}(x)$ | 0 | 6,25 | 8 | 6,75 | 4 | 1,25 | 0 | 1,75 | 8 | | N | | H | | W | | T | | | 5 |
| x | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | | | | | | | | | | | | | | | | | | | | | | | |
| $g_{25}(x)$ | 0 | 6,25 | 8 | 6,75 | 4 | 1,25 | 0 | 1,75 | 8 | | | | | | | | | | | | | | | | | | | | | | | |
| | N | | H | | W | | T | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3.1 | $g_{25}(x) = p(x)$ ergibt z.B. $\frac{1}{4}x^3 - \frac{5}{2}x^2 + \frac{29}{4}x - 5 = 0$; $x_1 = 5$ (Raten oder Taschenrechner) Mit Hilfe der Polynomdivision: $x_2 = 4$; $x_3 = 1$, also $S_1(5 0)$; $S_2(4 1,25)$; $S_3(1 8)$ | 7 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3.2 | (Graph siehe unten) | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3.3 | $A = \int_1^4 (g_{25}(x) - p(x)) dx = \left[\frac{1}{16}x^4 - \frac{5}{6}x^3 + \frac{29}{8}x^2 - 5x \right]_1^4 = \frac{45}{16}$ | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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| 4 | <p>Stetigkeit bei $x_0 = 5$ ist gegeben.</p> $h'(x) = \begin{cases} \frac{1}{4}(3x^2 - 18x + 15) & \text{für } x < 5 \\ \frac{1}{2}(x - 7) & \text{für } x > 5 \end{cases}$ <p>$\lim_{x \rightarrow 5^+} h'(x) = -1; \lim_{x \rightarrow 5^-} h'(x) = 0.$ h ist bei $x_0 = 5$ nicht differenzierbar.</p> | 4 |
| 5.1 | <p>$2400\pi = 2r^2\pi + 2r\pi h$; damit: $h = \frac{1200}{r} - r$</p> <p>$V = r^2\pi h \Rightarrow V(r) = \pi(1200r - r^3)$</p> | 4 |
| 5.2 | <p>$V'(r) = \pi(1200 - 3r^2); V''(r) = -6r\pi$</p> <p>$V'(r) = 0 \Rightarrow r = 20$, da $D_V = [12; 30]$</p> <p>$V''(20) < 0 \Rightarrow$ rel. Max. von V bei $r = 20$:</p> <p>$V(12) \approx 39810; V(20) \approx 50265; V(30) \approx 28274$</p> <p>damit: absolutes Maximum von V bei $r = 20$.</p> | 5 |
| | <p>Graph zu 2.6 und 3.2:</p> | |