AII Lösung

1. $f_a(x) = a(x^3 + 2x^2 - 7x + 4)$

1.1. $x_1 = 1$

$$(x^3 + 2x^2 - 7x + 4) : (x - 1) = x^2 + 3x - 4$$
$$-(x^3 - x^2)$$
$$3x^2 - 7x$$
$$-(3x^2 - 3x)$$
$$-4x + 4$$
$$-(-4x + 4)$$
$$/$$

$$x_{2/3} = \frac{-3 \pm \sqrt{9 + 4 \cdot 4}}{2}$$

$$x_{2/3} = \frac{-3 \pm 5}{2}$$

$x_2 = 1$ $x_3 = -4$
(doppelt) (einfach)

$$f_a(x) = a(x-1)^2(x+4)$$

bei reiner Angabe der NS
- pro richtigen NS 1 P
- pro Vielfachheit ½ P

6

1.2. $f_a'(x) = a(3x^2 + 4x - 7)$

$$f_a'(-3) = a(3(-3)^2 + 4 \cdot (-3) - 7) = 8a$$
$$f_a(-3) = a((-3)^3 + 2 \cdot (-3)^2 - 7 \cdot (-3) + 4) = 16a$$

$$y_T = mx + t$$
$$16a = 8a(-3) + 5$$
$$40a = 5$$
$$a = \frac{1}{8}$$

5

2.0. $f(x) = \frac{1}{8}(x^3 + 2x^2 - 7x + 4)$

2.1. $f'(x) = \frac{1}{8}(3x^2 + 4x - 7)$

$f'(x) = 0 \Rightarrow 3x^2 + 4x - 7 = 0$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 + 4 \cdot 3 \cdot 7}}{2 \cdot 3} = \frac{-4 \pm 10}{6}$$

$x_1 = 1$
$x_2 = -\frac{7}{3}$

Skizze zu $G_{f'}$

x	$x <$	$-\frac{7}{3}$	$< x <$	1	$< x$
$f'(x)$	$+$	0	$-$	0	$+$ ✓
G_f	↗	HOP ✓	↘	TIP ✓	↗ ✓

G_f sms für $x \in]-\infty; -\frac{7}{3}]$ ✓

G_f smf für $x \in [-\frac{7}{3}; 1]$ ✓

G_f sms für $x \in [1; +\infty[$

$f(1) = 0 \Rightarrow TIP(1|0)$

$f(-\frac{7}{3}) = \frac{125}{54} = 2{,}31$ ✓

$\Rightarrow HOP(-\frac{7}{3}|\frac{125}{54})$

7

2.2. $f''(x) = \frac{1}{8}(6x+4)$ ✓

$f''(x) = 0 \qquad 6x+4 = 0$ ✓

$x = -\frac{2}{3}$ ✓

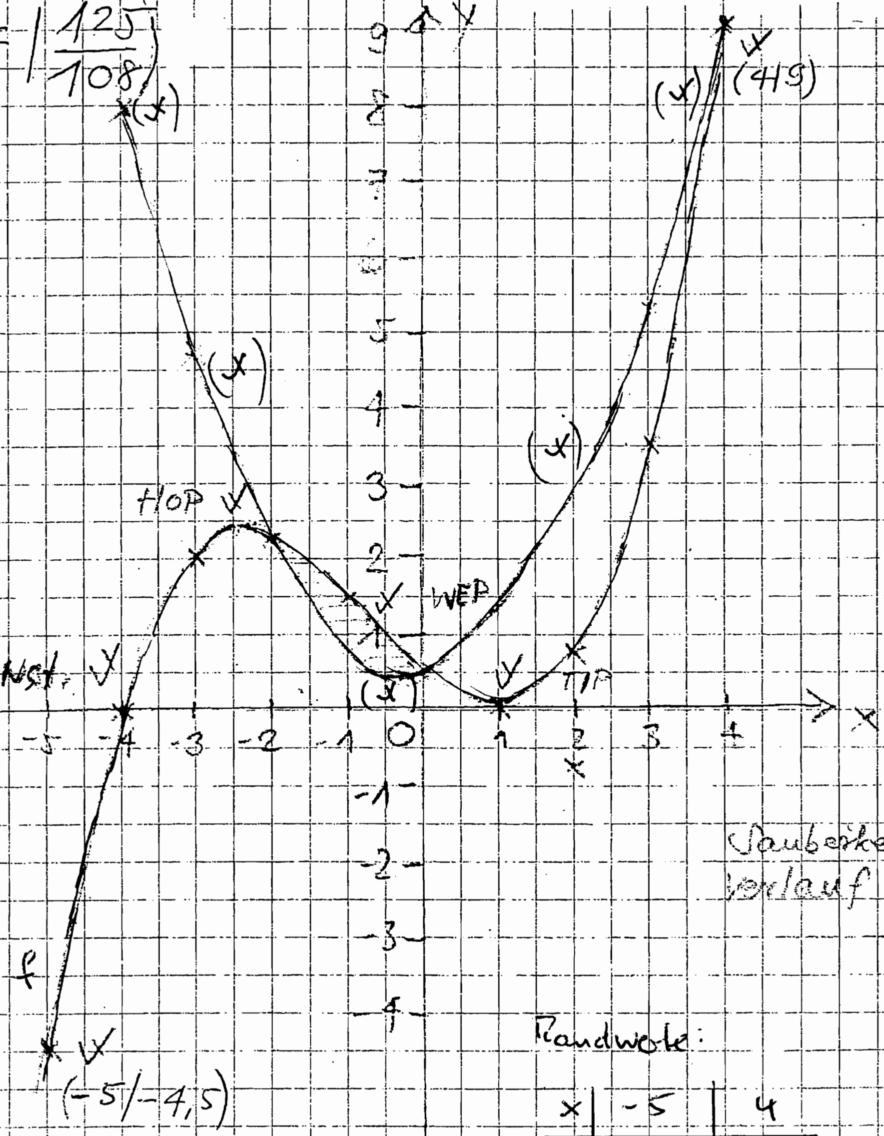
WEP existiert, da $x = -\frac{2}{3}$ einfache Nullstelle. ✓

$f(-\frac{2}{3}) = \frac{125}{108} = 1{,}157$ ✓

4

$P(-\frac{2}{3}|\frac{125}{108})$

2.3.



Sauberkeit ✓
Verlauf ✓

4

Randwerte:

x	-5	4
y	$-4{,}5$	9

3.1. $p''(x) = 1$

$p'(x) = x + c$

$p(x) = \frac{1}{2}x^2 + cx + d$

$A \in p: \quad 8 = \frac{1}{2}\cdot(-4)^2 + c\cdot(-4) + d$

$S \in p': \quad 0 = -\frac{1}{8} + c \quad \Rightarrow \quad c = \frac{1}{8}$

$8 = 8 + \frac{1}{8}\cdot(-4) + d \quad \Rightarrow \quad d = +\frac{1}{2}$

$\Rightarrow p(x) = \frac{1}{2}x^2 + \frac{1}{8}x + \frac{1}{2}$

$p(x) = ax^2 + bx + c$

$p'(x) = 2ax + b$

$p''(x) = 2a$

$2a = 1 \Rightarrow a = \frac{1}{2}$

$p'(-\frac{1}{8}) = -\frac{1}{4}\cdot\frac{1}{2} + b = 0$

$b = \frac{1}{8}$

$p(-4) = 16a - 4b + c = 8$

$8 - \frac{1}{2} + c = 8$

$c = \frac{1}{2}$

5

$p(x) = \frac{1}{2}x^2 + \frac{1}{8}x + \frac{1}{2}$

3.2. $f(x) = p(x)$

$\frac{1}{8}(x^3 + 2x^2 - 7x + 4) = \frac{1}{2}(x^2 + \frac{1}{4}x + 1) \quad | \cdot 8$

$x^3 + 2x^2 - 7x + 4 = 4x^2 + x + 4$

$x^3 - 2x^2 - 8x = 0$

$x(x^2 - 2x - 8) = 0$

$x_1 = 0 \qquad p(0) = +\frac{1}{2} \qquad S_1(0|+\frac{1}{2})$

$x_{2/3} = \frac{2 \pm \sqrt{4 + 4\cdot 8}}{2} = \frac{2 \pm 6}{2}$

$x_2 = 4 \qquad p(4) = 9 \qquad S_2(4|9)$

$x_3 = -2 \qquad p(-2) = 2{,}25 \qquad S_3(-2|2{,}25)$ 9

Zeichnung

3.3. $p(x) - f(x) > 0$ bzw. $p(x) > f(x)$

für $x \in]-\infty; -2[\cup]0; 4[$

In diesen Bereichen verläuft G_p oberhalb von G_f. 2

3.4. $A = \int_{-2}^{0} [f(x) - p(x)]\,dx = \int_{-2}^{0} (\frac{1}{8}x^3 - \frac{1}{4}x^2 - x)\,dx$

$= \left[\frac{1}{32}x^4 - \frac{1}{12}x^3 - \frac{1}{2}x^2\right]_{-2}^{0} = 0 - \left[\frac{1}{32}\cdot(-2)^4 - \frac{1}{12}(-2)^3 - \frac{1}{2}(-2)^2\right]$

$= 0 - (-\frac{5}{6}) = \frac{5}{6}$ FE

Markierung

5

4.0. $g(x) = -x^2 + 8x$

4.1. $V(r,h) = \frac{1}{3} r^2 \pi \cdot h$ ✓ NB: $h = -r^2 + 8r$ ✓

(3) $V(r) = \frac{1}{3} r^2 \cdot \pi (-r^2 + 8r)$ ✓ $= -\frac{1}{3}\pi r^4 + \frac{8}{3}\pi r^3$ ✓

4.2. $r > 0$ und $h > 0 \Rightarrow -r^2 + 8r > 0$ ✓

$r(-r+8) > 0$

$-r + 8 > 0$

$r < 8$

(3) $D_r =]0; 8[$ ✓ ✓

4.3. $V'(r) = -\frac{4}{3}\pi \cdot r^3 + 8\pi r^2$ ✓

$V''(r) = -4\pi \cdot r^2 + 16\pi r$ ✓

$V'(r) = 0$ ✓ $\Rightarrow \pi r^2 (-\frac{4}{3} r + 8) = 0$ ✓ $r_{1/2} = 0$ ✓ $\notin D$

$-\frac{4}{3} r = -8$ $r_3 = 6$ ✓ (ggf ✓ auf r_3)

$V''(6) = -48\pi < 0$ ✓ $\Rightarrow$ Max bei $r = 6$ ✓

$V(6) = -\frac{1}{3}\pi \cdot 6^4 + \frac{8}{3}\pi \cdot 6^3 = 144\pi = 452{,}4$ ✓

Das maximale Volumen beträgt 452,4 VE.

(7) $y_B = g(6) = -6^2 + 8 \cdot 6 = 12$ ✓ $B(6|12)$

Alternative zu 3.1

$p(x) = a\left(x+\frac{1}{8}\right)^2 + b$ ✓

$p''(x) = 1$

$p'(x) = x + c$ ✓

$p(x) = \frac{1}{2}x^2 + cx + d$ ✓ $\Rightarrow a = \frac{1}{2}$ ✓

$p(-4) = 8 \Rightarrow a\left(-\frac{31}{8}\right)^2 + b = 8$

$\frac{1}{2} \cdot \frac{31^2}{8^2} + b = 8$ ✓

$b = 8 - \frac{961}{128} = \frac{63}{128}$ ✓

$\Rightarrow p(x) = \frac{1}{2}\left(x+\frac{1}{8}\right)^2 + \frac{63}{128}$ ✓

$= \frac{1}{2}x^2 + \frac{1}{8}x + \frac{1}{2}$

(allgemeine Form nicht verlangt!)

5 P