12. Klasse	3. Schulaufgabe	2010/11

Musterlösung

Gruppe: A **Arbeitszeit: 85 Minuten** **BE: 47**

Analysis **14.2.2011**

1.1
$$f(x) = ax^3 + bx^2 + cx + d$$
$$f'(x) = 3ax^2 + 2bx + c$$
$$f''(x) = 6ax + 2b$$

$$I.\quad f(0) = -3 \Rightarrow d = -3$$
$$II.\quad f(2) = -1 \Rightarrow 8a + 4b + 2c = 2$$
$$III.\quad f'(2) = 0 \Rightarrow 12a + 4b + c = 0$$
$$IV.\quad f''(2) = 0 \Rightarrow 12a + 2b = 0$$

$$2\cdot III - II = V: \quad 16a + 4b = -2$$

$$V - 2\cdot IV = VI: \quad -8a = -2 \Rightarrow a = \frac{1}{4} \text{ in } IV$$

$$\Rightarrow b = -\frac{3}{2};\ a, b \text{ in } III \Rightarrow c = 3$$

$$\Rightarrow f(x) = \frac{1}{4}x^3 - \frac{3}{2}x^2 + 3x - 3$$ 10 BE

1.2 $g(x) = -1$ 1 BE

1.3
$$\int_0^2 (g(x) - f(x))dx = \int_0^2 \left(-\frac{1}{4}x^3 + \frac{3}{2}x^2 - 3x + 2\right)dx = \left[-\frac{1}{16}x^4 + \frac{1}{2}x^3 - \frac{3}{2}x^2 + 2x\right]_0^2 = -1 + 4 - 6 + 4 - 0 = 1$$ 5 BE

$\Rightarrow A = 1\,FE$

1.4

c	3	4	5
Integral	< 0	$= 0$	> 0

3 BE

2.1 $A(x) = (2 + x)\left(4 - \frac{1}{2}x\right) = -\frac{1}{2}x^2 + 3x + 8, \quad 0 < x < 6$ 5 BE

2.2 $A'(x) = -x + 3 = 0 \Rightarrow x = 3 \in D(A)$

$A''(x) = -1 < 0 \Rightarrow$

Für x = 3 wird der Inhalt der Rechtecksfläche maximal.

$A_{max} = A(3) = 12{,}5\left[cm^2\right]$ 4 BE