

Lösung 3. SA Analysis Gruppe A

1a)  $\mathbb{D}_{f_1} = \mathbb{D}_{f_2} = \mathbb{R}$  ✓  
 $f_1(4) = 0,5(4^3 - 6 \cdot 4^2 + 9 \cdot 4 - 4) = 0$  ✓  
 $f_2(4) = -4^2 + 12 \cdot 4 - 32 = 0$  ✓ ✓  $\Rightarrow f_1(4) = f_2(4)$   
 $f$  ist stetig bei  $x=4$  ✓

b)  $f_1'(x) = 0,5(3x^2 - 12x + 9)$

$f_2'(x) = -2x + 12$

$f_1'(4) = 0,5(3 \cdot 4^2 - 12 \cdot 4 + 9) = 4,5$  ✓  $\Rightarrow f_1'(4) \neq f_2'(4)$

$f_2'(4) = -2 \cdot 4 + 12 = 4$  ✓

✓  $\Rightarrow f$  ist bei  $x=4$  nicht differenzierbar ✓

$\Rightarrow G_f$  hat bei  $x=4$  einen Knick ✓

(4P)

2.  $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$  ✓

$f''(x) = 6ax + 2b$  ✓

$f(0) = -1$  ✓  $\Rightarrow d = -1$  ✓

$f(2) = 0$  ✓  $8a + 4b + 2c - 1 = 0$  ✓

$f'(0) = 4,5$  ✓  $c = 4,5$  ✓

$f''(2) = 0$  ✓  $12a + 2b = 0$  ✓

$8a + 4b = -8$

$12a + 2b = 0$   $\cdot (-2)$

$8a + 4b = -8$   
 $-24a - 4b = 0$   $\cdot (-1)$

$-16a = -8$

$a = 0,5$  ✓

$8 \cdot 0,5 + 4b = -8$

$4b = -12$

$b = -3$  ✓

$f(x) = 0,5x^3 - 3x^2 + 4,5x - 1$  ✓

(9P)

3.1.  $L = \pi r + 2r + 2a = 60$  ✓

$a = \frac{60 - \pi r - 2r}{2} = 30 - r\left(\frac{\pi}{2} + 1\right)$  ✓

$A(r) = \frac{1}{2}\pi r^2 + a \cdot 2r$  ✓

$= \frac{1}{2}\pi r^2 + (30 - r\left(\frac{\pi}{2} + 1\right)) \cdot 2r$  ✓

$= \frac{1}{2}\pi r^2 + 60r - 2r^2\left(\frac{\pi}{2} + 1\right)$

$= \frac{1}{2}\pi r^2 + 60r - \pi r^2 - 2r^2$

$= -r^2\left(2 + \frac{\pi}{2}\right) + 60r$  ✓

(5P)

3.2.  $A'(r) = -2\left(2 + \frac{\pi}{2}\right)r + 60$  ✓

$-(4 + \pi)r + 60 = 0$  ✓  $\Rightarrow r = \frac{60}{4 + \pi} \approx 8,4$  ✓

$A''(r) = -(4 + \pi) < 0$   $\Rightarrow$  Maximum bei  $r = 8,4$  ✓

Länge/Kopf:  $a = 30 - 8,4\left(\frac{\pi}{2} + 1\right) = 8,4$  ✓

(5P)