Lösung:

1.1
$f(x) = ax^4+bx^3+cx^2+dx+e$
$f'(x) = 4ax^3+3bx^2+2cx+d$
$f''(x)= 12 ax^2+6bx+2c$

Terrassenpunkt (0/0) $f(0) = 0 \Rightarrow e=0$
$f'(0) =0 \Rightarrow d=0$
$f''(0) = 0 \Rightarrow c=0$
P(2/8) $f(2)=8 \Rightarrow 16a+8b = 8$ (I)
horizontale Tangente $\Rightarrow$ $32a +12b = 0$ (II)

II-2*I $-4b = -16 \Rightarrow b= 4$ einsetzen in I: $16a+32 = 8 \Rightarrow a=-1,5$

1.2 $f''(2) < 0$ also HOP

2.1 V = a b h da a = 3b $\Rightarrow$ V= 3b b h = 3 b² h

Nebenbedingung : a b + 2 a h + 2 b h = 20 000 also 3b² + 6 b h + 2 b h = 20 000

$$3b^2 + 8bh = 20\,000$$
$$\Rightarrow \quad h = \frac{20000-3b^2}{8b}$$

Einsetzen in V : $V= 3b^2 \frac{20000-3b^2}{8b} = -\frac{9}{8}b^3+7500b$
Definitionsmenge: a= 0

$h=0 \Rightarrow 0=\frac{20000-3b^2}{8b}$ also $0 = 200000-3b^2 \Rightarrow b_1=\sqrt{\frac{20000}{3}}$; $b_2= -\sqrt{\frac{20000}{3}}$;

also $D_V = \left]0;\sqrt{\frac{20000}{3}}\right[$

2.2 V(40) = 228000 da gilt a = 3 b ist a = 120 cm und aus V= a b h folgt h = 47,5 cm

2.3 $V'(b) = -\frac{27}{8}b^2+7500$ und $V''(b) = -\frac{27}{4}b$
$V'(b) = 0$

also $0 = -\frac{27}{8}b^2+7500$ $b_1=\sqrt{\frac{20000}{9}}$ und $b_2=-\sqrt{\frac{20000}{9}}$ keine Lösung

$b_1=\sqrt{\frac{20000}{9}}$ $V''(b_1) <0$ also Maximum