Lösung – Analysis

1. $f(x) = -\frac{1}{81}\cdot(x^4 - 108x)$

1.1 $f(x)=0 \Rightarrow x^4 - 108x = 0 \Rightarrow x\cdot(x^3-108)=0$

③ $\Rightarrow x_1 = 0$ (einfach), $x_2 = \sqrt[3]{108} = 3\cdot\sqrt[3]{4} \approx 4{,}76$ (einfach)

1.2 $f'(x) = -\frac{1}{81}\cdot(4x^3-108) = 0$

⑥ $\Rightarrow 4x^3 - 108 = 0 \Rightarrow x^3 = 27 \Rightarrow x = 3$ (einfach)

Mon.ber.	$-\infty < x <$	3	$< x < \infty$
f'	+	0	−
G_f	steigt	HP	fällt

$f(3) = -\frac{1}{81}\cdot(81-324) = 3 \Rightarrow HP(3;3)$

1.3 $f''(x) = -\frac{1}{81}\cdot 12x^2 = 0 \Rightarrow x = 0$ (doppelt) $\Rightarrow$ kein WP !

③ oder:

Kr.ber.	$-\infty < x <$	0	$< x < \infty$
f''	−	0	−
G_f	rechts	FP kein WP	rechts

1.4 $f'(0) = -\frac{1}{81}\cdot(0-108) = \frac{108}{81} = \frac{4}{3}$

② also: $t: y = \frac{4}{3}\cdot x$

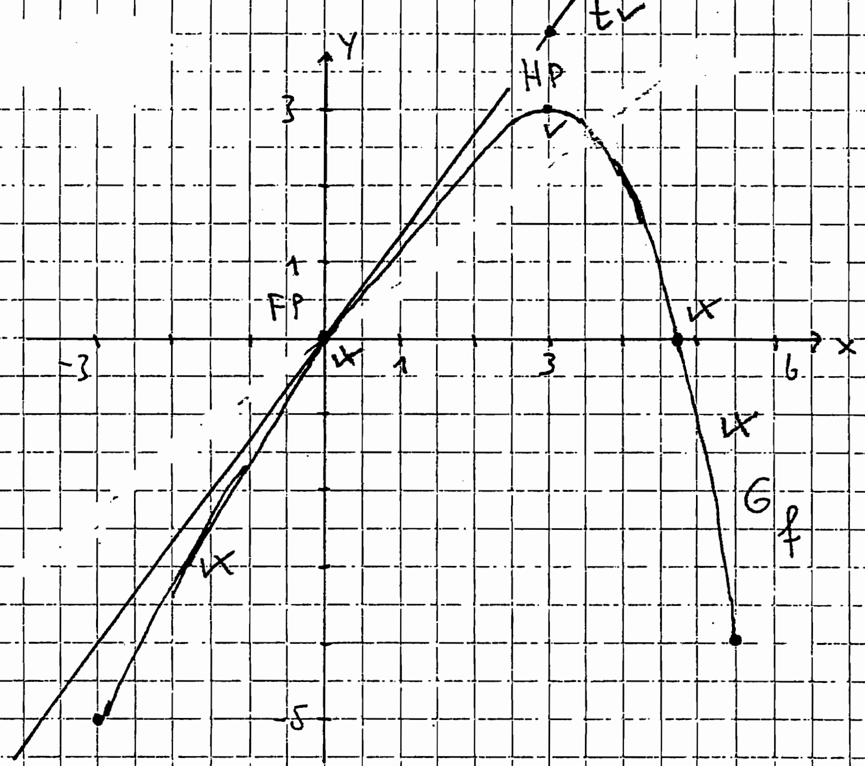
3.

④

Mon.ber.	$-\infty < x <$	−2	$< x <$	1	$< x < \infty$
f'	−	0	+	0	+
G_f	fällt	TP	steigt	TEP	steigt

1.5 $f(-3) = -\frac{405}{81} = -5$; $f(5,5) \approx -3,96$

(4)



2. $f_k(x) = x^3 - 3x^2 + kx + k^2$, $k \in \mathbb{R}$

2.1 P in f_k $\Rightarrow$ $f_k(-1) = -2$ $\Rightarrow$ $-1 - 3 - k + k^2 = -2$

(4)

$\Rightarrow$ $k^2 - k - 2 = 0$ $\Rightarrow$ $k_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$ $\Rightarrow$ $k_1 = -1$

$k_2 = 2$

2.2 $f_k'(x) = 3x^2 - 6x + k$

(4) TEP falls Diskr. von $f_k' = 0$ $\Rightarrow$ $36 - 12k = 0$ $\Rightarrow$ $k = 3$

<u>oder:</u> $f_k''(x) = 0$ und $f_k'(x) = 0$

$f_k''(x) = 6x - 6 = 0$ $\Rightarrow$ $x = 1$

$f_k'(1) = 0$ $\Rightarrow$ $3 - 6 + k = 0$ $\Rightarrow$ $k = 3$