Lösung

1. $f(x) = ax^4 + bx^3 + cx^2 + dx + e$
$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$
$f''(x) = 12ax^2 + 6bx + 2c$

I. $f(0) = 0 \Rightarrow e = 0$
II. $f'(0) = 0 \Rightarrow d = 0$
III. $f''(0) = 0 \Rightarrow c = 0$
IV. $f(-3) = 9 \Rightarrow 81a - 27b = 9$
V. $f'(-3) = 0 \Rightarrow -108a + 27b = 0$ 9

IV + V: $-27a = 9 \Rightarrow a = -\frac{1}{3}$ in V $\Rightarrow b = -\frac{4}{3}$

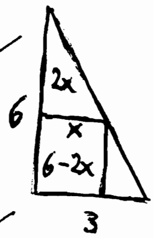
$\Rightarrow f(x) = -\frac{1}{3}x^4 - \frac{4}{3}x^3$

2. $f(x) = -\frac{1}{3}x^3(x+4) = 0 \Rightarrow x_1 = -4;\ x_2 = 0$

$A = \int_{-4}^{0} \left(-\frac{1}{3}x^4 - \frac{4}{3}x^3\right)dx = \left[-\frac{1}{15}x^5 - \frac{1}{3}x^4\right]_{-4}^{0} = 0 - \left(\frac{1024}{15} - \frac{256}{3}\right)$ 6

$\Rightarrow A = 17{,}07$ [FE]

3.1 Zimmergrundseite $= 2x$; Zimmerhöhe $= 6 - 2x$



$\Rightarrow V(x) = (2x)^2 \cdot (6 - 2x) = -8(x^3 - 3x^2)$

$D(V) =]0; 3[$ 5

3.2 $V'(x) = -8(3x^2 - 6x) = -8x(3x - 6) = 0$

$\Rightarrow x_1 = 0 \notin D(V);\ x_2 = 2 \in D(V)$ 7

$V''(x) = -8(6x - 6)$

$V''(2) = -48 < 0 \Rightarrow$ Für $x = 2$ wird das Zimmervolumen maximal.

3.3 $V(\text{Zimmer}) = V(2) = 32\ [m^3]$; $V(\text{Pyramide}) = \frac{1}{3} \cdot 6^2 \cdot 6 = 72\ [m^3]$

$\Rightarrow \frac{32}{72} \cdot 100\,\% = 44{,}4\,\%$ des Dachvolumens wird durch das Zimmer genutzt. 3

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