Lösung 3. Schulaufg. Analysis

1.1 $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$f''(x) = 12ax^2 + 6bx + 2c$

$f'(0) = 0 \Rightarrow d = 0$

$f''(0) = 0 \Rightarrow c = 0$

$f'(3) = 0 \Rightarrow 108a + 27b = 0 \quad | :27 \quad 4a + b = 0 \quad b = -4a$ ✓

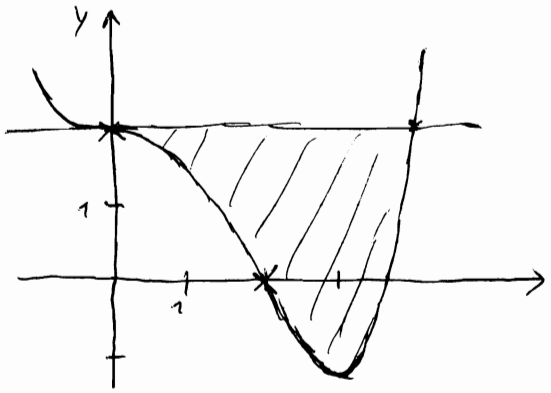
$f'(2) = -2 \Rightarrow 32a + 12b = -2 \quad 32a - 48a = -2$

$f(2) = 0 \Rightarrow 16a + 8b + e = 0 \quad -16a = -2 \quad a = \frac{1}{8}$ ✓

$16 \cdot \frac{1}{8} + 8 \cdot \left(-\frac{1}{2}\right) + e = 0 \Rightarrow e = 2$ ✓ $\quad \Rightarrow b = -\frac{1}{2}$ ✓

also $f(x) = \frac{1}{8}x^4 - \frac{1}{2}x^3 + 2$ ✓

1.2



1.3 $\frac{1}{8}x^4 - \frac{1}{2}x^3 + 2 = 2 \quad x^3\left(\frac{1}{8}x - \frac{1}{2}\right) = 0$ ✓ (oder ins Werfetab.)

$x_1 = 0 \quad x_2 = 4$ ✓

$\int_0^4 \left(\frac{1}{8}x^4 - \frac{1}{2}x^3\right) dx = \frac{1}{40}x^5 - \frac{1}{8}x^4 \Big|_0^4$ ✓

$= \frac{1}{40} \cdot 4^5 - \frac{1}{8} \cdot 4^4 - 0 = -6{,}4$ ✓ ✓

$A = 6{,}4$ FE ✓

2.1 $V = \frac{1}{3} G H = \frac{1}{3} \cdot \frac{\sqrt{3}}{4} a^2 \cdot H$ ✓ ✓ $\quad a + H = 30 \Rightarrow H = 30 - a$ ✓

$V(a) = \frac{\sqrt{3}}{12} a^2 (30 - a) = \frac{\sqrt{3}}{12}\left(30a^2 - a^3\right)$ ✓

mit $0 < a < 30$ ✓

2.2 $V'(a) = \frac{\sqrt{3}}{12}\left(60a - 3a^2\right) = 0 \Rightarrow 3a(20 - a) = 0$ ✓

$(a_1 = 0) \quad a_2 = 20$ ✓

$V''(a) = \frac{\sqrt{3}}{12}(60 - 6a)$ ✓ $\quad V''(20) = \frac{\sqrt{3}}{12}(60 - 120) < 0 \Rightarrow$ Max. ✓

$V(20) = \frac{\sqrt{3}}{12} \cdot 400 \cdot 10 \approx 577{,}35\,cm^3$ ✓