Lösung – Analysis – Gruppe A

1. $f(x) = 2 \cdot (\ln x)^2 - 2$, $D_f = \mathbb{R}^+$

1.1 $\lim\limits_{x \to \infty} \left(2 \cdot \underset{\to \infty}{(\ln x)^2} - 2 \right) = \infty$ ✓

(2)

$\lim\limits_{x \to 0} \left(2 \cdot \underset{\to \infty}{(\ln x)^2} - 2 \right) = \infty$ ✓ ($\Rightarrow$ vert. Asy.: $x = 0$)

1.2 $f(x) = 0 \Rightarrow 2 \cdot (\ln x)^2 - 2 = 0 \Rightarrow (\ln x)^2 \overset{\text{LK}}{=} 1$

(3)

$\Rightarrow \ln x \overset{\text{LK}}{=} \pm 1 \Rightarrow \underline{x_1 \overset{\text{✓}}{=} e^{-1}}$, $\underline{x_2 \overset{\text{✓}}{=} e}$

1.3 $f'(x) = 4 \cdot \ln x \overset{\text{✓}}{\cdot} \frac{1}{x} = 4 \cdot \frac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \Rightarrow \underline{x \overset{\text{✓}}{=} 1}$;

(5)

$f''(x) \overset{\text{✓}}{=} 4 \cdot \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = 4 \cdot \frac{1 - \ln x}{x^2}$

$f''(1) = 4 \cdot \frac{1-0}{1} = 4 \overset{\text{LK}}{>} 0$

$f(1) \overset{\text{✓}}{=} -2$

$\Rightarrow \underline{\overset{\text{LK}}{\text{TP}}(1; -2)}$

1.4 • $f''(x) = 4 \cdot \frac{1 - \ln x}{x^2} = 0 \Rightarrow 1 - \ln x \overset{\text{✓}}{=} 0$

(6)

$\Rightarrow \ln x = 1 \Rightarrow x \overset{\text{✓}}{=} e$, einfach also Wendestelle

$f(e) = 0 \Rightarrow \underline{\text{WP}\overset{\text{✓}}{(e; 0)}}$

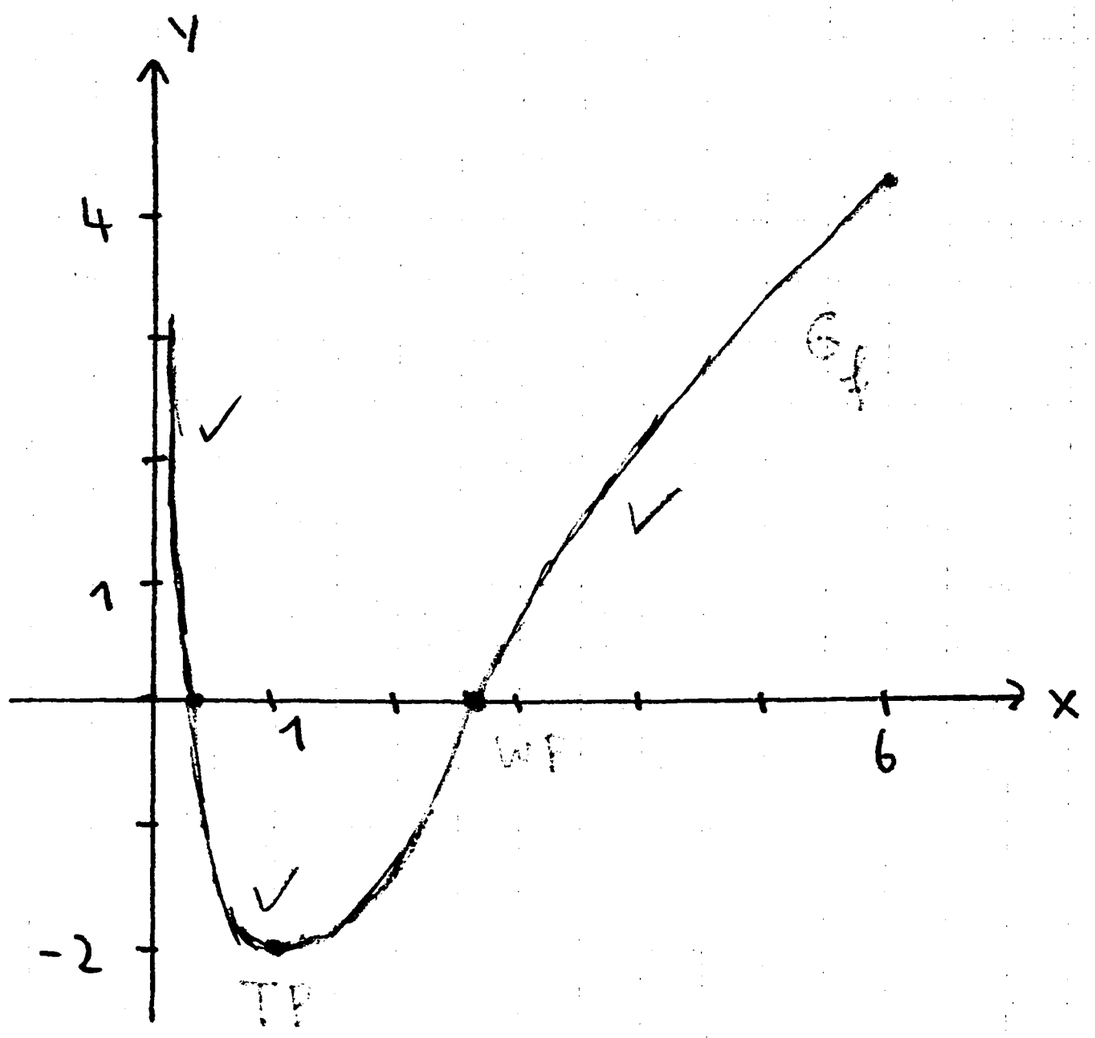
• $t: y = f'(e) \cdot x + a = \frac{4}{e} x \overset{\text{✓}}{+} a$

WP in $t \Rightarrow 0 = 4 + a \Rightarrow a \overset{\text{✓}}{=} -4$

also $\underline{t: y \overset{\text{✓}}{=} \frac{4}{e} x - 4}$;

1.5 $f(6) \approx 4{,}42$ ✓

(4)



1.6 $\int (\ln x)^2 \, dx = x \cdot (\ln x)^2 - 2x \cdot \ln x + 2x + c, \; c \in \mathbb{R}$

(4)

$$A = \left| \int_{e^{-1}}^{e} f(x) \, dx \right| = \left| \left[2 \cdot x \cdot (\ln x)^2 - 4x \cdot \ln x + 4x - 2x \right]_{e^{-1}}^{e} \right| =$$

$$= \left| \left[2x \cdot (\ln x)^2 - 4x \cdot \ln x + 2x \right]_{e^{-1}}^{e} \right| =$$

$$= \left| 2e - 4e + 2e - \left(2e^{-1} + 4e^{-1} + 2e^{-1} \right) \right| = \underline{\underline{8e^{-1} \text{ FE } (\approx 2{,}94)}}$$

2. $v = v_0 + S \cdot (\ln m_0 - \ln m)$, $v_0 = 28840 \frac{km}{h}$, $v = 40320 \frac{km}{h}$

2.1 $m = 42 - 32 = 10$ Tonnen ✓

(3) $40320 = 28840 + S \cdot (\ln 42 - \ln 10)$ ✓

$$\Rightarrow S = \frac{40320 - 28840}{\ln 42 - \ln 10} \approx \underline{\underline{8000}} \checkmark$$

2.2 $34000 = 28840 + 8000 \cdot (\ln 42 - \ln m)$ ✓

(4)

$$\Rightarrow \ln 42 - \ln m = \frac{34000 - 28840}{8000} = 0{,}645 \checkmark$$

$$\Rightarrow \ln m = \ln 42 - 0{,}645 \approx 3{,}09 \Rightarrow m = 22 \checkmark$$

$42 - 22 = 20$ Tonnen wurden verbrannt! ✓